Calculus Warm Up #4-1

Use the trapezoid rule to approximate the area between the curve and the x-axis represented by the integral.

$$\int_1^7 \frac{\sqrt{x-1}}{x}\,dx \quad, n = 6$$

HW Questions: p. 278

In Exercises 5–28, evaluate the indefinite integral and check the result by differentiation.

5. $\int (1 + 2x)^4(2)\,dx$

7. $\int \sqrt{9 - x^2}(-2x)\,dx$

9. $\int x^2(x^3 - 1)^4\,dx$

11. $\int 5x\sqrt[3]{1 - x^2}\,dx$ let $u = 1 - x^2$ $du = -2x\,dx$

$-\frac{5}{2}\int -2x\,(1-x^2)^{1/3}\,dx$

$-\frac{5}{2}\int u^{1/3}\,du$

13. $\displaystyle\int \frac{x^2}{(1 + x^3)^2}\,dx$

15. $\displaystyle\int \frac{4x}{\sqrt{16 - x^2}}\,dx$

17. $\displaystyle\int \frac{x + 1}{(x^2 + 2x - 3)^2}\,dx$

19. $\displaystyle\int \left(1 + \frac{1}{t}\right)^3 \left(\frac{1}{t^2}\right) dt$

21. $\displaystyle\int \frac{1}{\sqrt{2x}}\,dx$

23. $\displaystyle\int \frac{x^2 + 3x + 7}{\sqrt{x}}\,dx = \int \left(x^{3/2} + 3x^{1/2} + 7x^{-1/2}\right) dx$

$$= \frac{2x^{5/2}}{5} + 2\cdot\frac{\cancel{3}x^{3/2}}{\cancel{3}} + 2\cdot\frac{7x^{1/2}}{1} + C$$

$$= \frac{2}{5}x^{5/2} + 2x^{3/2} + 14x^{1/2} + C$$

25. $\displaystyle\int t^2\left(t - \frac{2}{t}\right) dt$

27. $\displaystyle\int (9 - y)\sqrt{y}\,dy$

$$\int \left(9y^{1/2} - y^{3/2}\right) dy$$

5.5 Integration Techniques

Pattern Recognition

Change of Variables

U-Substitution (with pattern recognition)

Today:

U-Substitution (when the integrand doesn't have the variable part of du.)

The General Power Rule for Integration

Definite Integrals

Integration of Even and Odd Functions

Many integrands have the essential variable part of du, and just need adjusting for the constant multiple.

Let $u = 2x^2 + 5$

$du = 4x\,dx$

Example:

$$\frac{1}{4}\int 4x(2x^2+5)^3\,dx$$

What if it doesn't have the variable part of du?

$$\int (2x^2+5)^3\,dx \neq \frac{1}{4x}\int (2x^2+5)^3\,4x\,dx$$

Change of variable, U-Substitution extended...

Evaluate $\int x\sqrt{2x-1}\,dx$.

let $u = 2x-1$ $\quad du = 2\,dx$

$x = \frac{u+1}{2}$

$$= \frac{1}{2}\int x\underbrace{\sqrt{2x-1}}_{\sqrt{u}}\;\underbrace{2\,dx}_{du}$$

$$= \frac{1}{2}\int \frac{(u+1)}{2}\,u^{-1/2}\,du$$

$$= \frac{1}{4}\int \left(u^{1/2} + u^{-1/2}\right)du \longrightarrow \frac{1}{\cancel{4}\,2}\left[\frac{\cancel{2}u^{3/2}}{3} + \frac{\cancel{2}u^{1/2}}{1}\right] + C$$

$$= \frac{(2x-1)^{3/2}}{6} + \frac{(2x-1)^{1/2}}{2} + C$$

The General Power Rule for Integration

If g is a differentiable function of x, then

$$\int [\underbrace{g(x)}_{u}]^n \underbrace{g'(x)\,dx}_{du} = \frac{[g(x)]^{n+1}}{n+1} + C, \quad n \neq -1.$$

Equivalently, if $u = g(x)$, then

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad \boxed{n \neq -1.}$$

* We've seen this before. The point here is to notice that $n \neq -1$

$$\int \frac{1}{x}\,dx = \int x^{-1}\,dx$$

$$= \ln|x| + C$$

* Power Rule does not apply here!

Definite Integrals. The limits of integration must agree with your variable of integration.

After you integrate, but before you evaluate the integral, change the limits for u, or change back to x first.

let $u = x^2 + 1$ $du = 2x\,dx$

for $x = 0 \rightarrow u = 1$

$x = 1 \rightarrow u = 2$

Evaluate

$$\frac{1}{2}\int_0^1 2x(x^2+1)^3\,dx.$$

$$\frac{1}{2}\int_1^2 u^3\,du = \frac{1}{2}\left[\frac{u^4}{4}\right]_1^2 = \frac{1}{2}\left[4 - \frac{1}{4}\right]$$

$$= \frac{15}{8}$$

You try:

Evaluate

$$A = \frac{1}{2}\int_1^5 \frac{2x}{\sqrt{2x-1}}\,dx.$$

let $u = 2x - 1$ $\quad du = 2\,dx$

$x = \frac{u+1}{2}$

for $x = 1$
$u = 1$

for $x = 5$
$u = 9$

$$A = \frac{1}{2}\int_1^9 u^{-1/2}\left(\frac{u+1}{2}\right) du$$

$$\frac{1}{4}\int_1^9 \left(u^{1/2} + u^{-1/2}\right) du$$

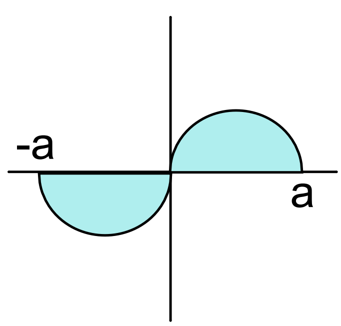
$$\frac{1}{4}\left[\frac{2u^{3/2}}{3} + \frac{2u^{1/2}}{1}\right]_1^9 = \frac{16}{3}$$

Remembering Odd and Even Functions

Odd

$f(-x) = -f(x)$

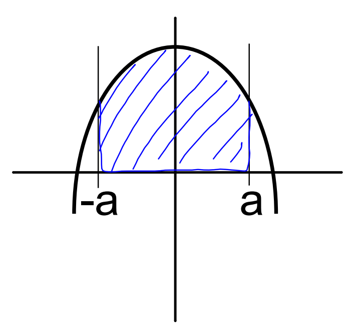
origin symmetry



Even

$f(-x) = f(x)$

y-axis symmetry



THEOREM 5.17 INTEGRATION OF EVEN AND ODD FUNCTIONS

Let f be integrable on the closed interval $[-a, a]$.

1. If f is an *even* function, then

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx.$$

2. If f is an *odd* function, then

$$\int_{-a}^{a} f(x)\,dx = 0.$$

EXAMPLE 10 Integration of an odd function → confirm odd

$f(-x) = -f(x)$

Evaluate

$$\int_{-2}^{2} (x^5 - 4x^3 + 6x)\,dx.$$

$(-x)^5 - 4(-x)^3 + 6(-x)$

$-x^5 + 4x^3 - 6x$

$-(x^5 - 4x^3 + 6x)$ ✓

$= 0$

HW: p. 278 # 29 - 51 odd

#35 hint: use conjugate!

HW Week 3 tomorrow:

pgs. 267, 268, 278